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An Empirical Model Relating the Auroral Geomagnetic Activity to the Interplanetary Magnetic Field

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Abstract. A damped linear oscillator (LRC circuit) predicts a significant amount of the AL index variations when it is driven by the IMF's B_{South} component. The model parameters are determined from fitting the AL time series and show a small, apparently nonsystematic variation with the activity level. Following an isolated external disturbance the LRC model's response peaks at 15 min – 0.5 h and decays after 1–2 more hours. The time scales and the correlation between observed AL and the circuit output (60–80%) are in good agreement with earlier linear prediction filter results. The results support the modeling of the global magnetospheric behavior with dynamical systems of a few degrees of freedom.

Introduction

One of the long-standing goals of magnetospheric research is the prediction of the geomagnetic activity on the ground. Statistical studies (Baker in [Kamide and Slavin, 1986]) show that the solar wind and geomagnetic activities are closely related through the solar wind- magnetosphere coupling [Kamide and Slavin, 1986; McPherron, 1991]. The recurrent character of the magnetospheric events suggests that the coupling is deterministic rather than stochastic. The coupling is also believed to be global, i.e. to involve interactions over large regions of the magnetosphere, and it is eventually responsible for the magnetospheric storms and substorms. The nonlinear nature of the coupling has been shown by the inadequacy of linear filtering [Bargatze et al., 1985]. This nonlinearity combined with the erratic solar wind input gives rise to irregular geomagnetic fluctuations. Recently it was suggested that the nonlinear coupling may be effectively described by a *dynamical system*, or a small set of nonlinear ordinary differential equations [Baker et al., 1990]. This approach is based on the observation that complex physical systems, such as plasmas and fluids, can be described by relatively simple dynamics when they exhibit large-scale coherent behavior, or self-organization. In such cases the small number of degrees of freedom of the model is sufficient to describe the observed irregular large-scale variations. Spatial and temporal self-organization in the magnetospheric plasma are evident in the development of global events like storms and substorms.

The number of effective degrees of freedom necessary to model the auroral geomagnetic activity has been estimated to be a small quantity (3–4) in several studies [Vassiliadis et al., 1990; Roberts, 1991; Shan et al., 1991; Pavlos et al., 1992]. These results were questioned [Prichard and Price, 1992] on the basis of uncertainties of the technique used, most importantly the lack of stationarity of the time series analyzed. However, the findings of an improved technique [Sharma et al., 1993] supports the hypothesis of few degrees of freedom.

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This paper takes a different approach to the hypothesis of few degrees of freedom by examining the extent to which a small number of degrees of freedom (a "low-dimensional" model) reproduces the electrojet index variations.

The LRC Circuit Model

The magnetosphere is an open system and its activity depends on the energy flux "input" received from the solar wind in the recent past. The "output" is the observed global activity manifested when the system dissipates the accumulated energy. In practice the input is expressed in terms of solar wind variables while the output is the set of all quantities that can be measured or indirectly deduced, e.g: electrojet intensity, ionospheric conductivity, ring current flux, lobe field strength, plasma sheet temperature, etc. An output observable, for example the electrojet intensity, is a function of the recent history of the input which is called the *response* to the solar wind for the electrojet intensity. The responses for all magnetospheric quantities make up the observed *global magnetospheric response* to the solar wind. The response for a magnetospheric observable is approximated by an empirical model which is successively refined. By using the solar wind input as a driver and matching the model output to the observations this procedure yields the model parameters, similarly to the calculation of the above-mentioned linear filters.

The empirical model presented below seeks to reproduce the observed variations of the westward electrojet intensity for a given interplanetary magnetic field (IMF) input. The electrojet variations are represented by the AL index obtained from the North-South magnetic field fluctuations measured at 11 stations around the northern auroral zone. Some limitations of AL in estimating the westward electrojet have been discussed in several articles (Baumjohann in [Kamide and Slavin, 1986]; [Kroehl, 1989]). The solar wind variations are represented by IMP-8 measurements of the Southward IMF component, B_{South}, which triggers most of the activity in the magnetotail and corresponding AL intensifications. The cross-correlation between B_{South} and AL peaks at 50–60% about 1 h after the Southward field reaches the magnetopause [McPherron, 1991].

The magnetosphere is modeled as a driven linear oscillator (an LRC circuit —Fig. 1) which is supercritically damped to model the undisturbed magnetosphere's return to a quiet "ground" state [Baker et al., 1990]. A damped oscillator is the common basis for several recent nonlinear dynamical models of the global magnetosphere [Goertz et al., 1993; Klimas et al., 1992; Weimer, 1991]. The input voltage V is directly proportional to the Southward IMF, while the circuit output, the resistor current i_R, is compared to the observed AL. The model equations are

$$\begin{aligned} \frac{di_L}{dt} &= \frac{1}{L}(V - v_C) \\ \frac{dv_C}{dt} &= \frac{1}{C}\left(i_L - \frac{v_C}{R}\right) \end{aligned} \qquad (2.1)$$

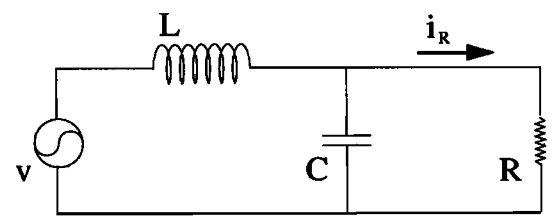

Figure 1 The linear LRC model is driven by a voltage V proportional to the Southward IMF and its output, i_R, is compared to the electrojet AL index.

where the dynamical variables are the inductance current i_L and the capacitor voltage v_C-i_R.

In each run the parameters L, R, and C are held fixed and the circuit is driven by a given B_{South} time series from a database of long (7–10 d) and short (1–2 d) intervals described in [Clauer et al., 1983]. After (2.1) has been integrated, the output time series $\widehat{AL}(t) \equiv i_R(t)$ is compared to the AL observations. The LRC values are iteratively adjusted to minimize the difference between prediction and observation

$$\chi^2(L,R,C) = \frac{1}{N\sigma_{AL}^2} \sum_{i=1}^{N} \left(\widehat{AL}(t_i; L,R,C) - AL(t_i)\right)^2 \tag{2.2}$$

which is proportional to the shaded area in Fig. 2. The χ^2 is minimized using the downhill simplex method [Press et al., 1986]. Because of its dynamic range the χ^2 function is more sensitive to small changes in the LRC parameters than either the cross-correlation between AL and $\widehat{AL}$ or the "prediction efficiency" (defined in [Bargatze et al., 1985]) and it was chosen as the main figure of merit. Still, the cross-correlation and the prediction efficiency are more familiar so they will be used below to quantify the results.

The correlation and prediction efficiency indicate that the linear circuit can approximate the observed auroral AL activity to a good extent. Fig. 3 shows the two statistical quantities for several short (< 2 day) intervals. The intervals were selected from the data set if they were preceded and followed by a "quiet" (<100 nT) interval of two hours or longer. Many points fall within or above what has been considered a satisfactory performance of linear prediction filter analyses (80% correlation and 40% prediction efficiency

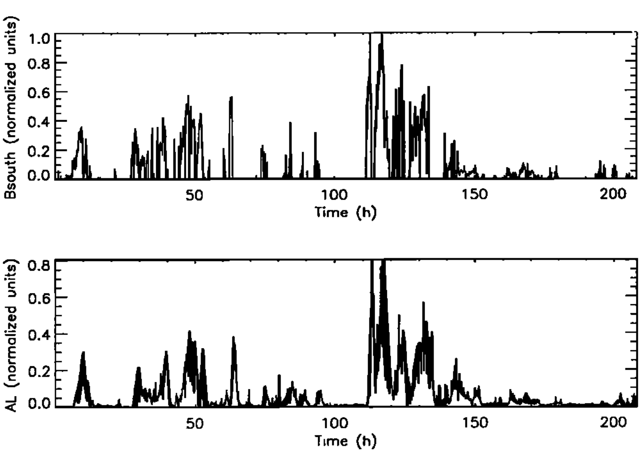

Figure 2 The linear circuit is driven by the solar wind B_{South} (upper panel), and reproduces the AL behavior to a good extent (lower panel). The shaded area in the lower panel is the absolute difference between observed and "predicted" AL.

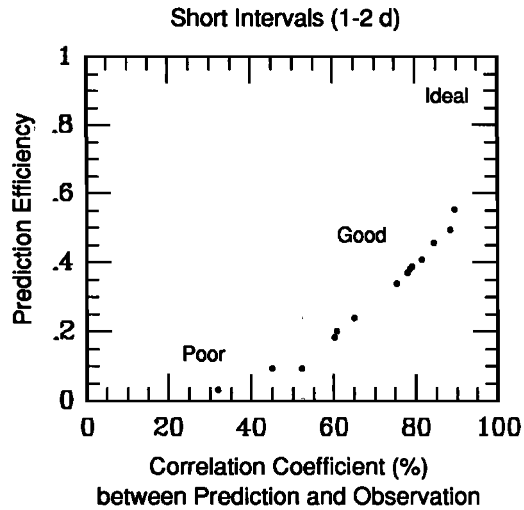

Figure 3 The cross-correlation between prediction and observation plotted versus the prediction efficiency.

[Bargatze et al., 1985] (Note, however, that McPherron and Blanchard [1991] have recently reported higher values for their linear prediction filters). Smoothing the 2.5-min data further increases the correlation and prediction efficiency.

A closer examination of the model's performance shows that it captures the long-timescale features of the observed time series down to a scale of 1 h or less. Finer features are not always resolved adequately and onset times may differ by 15 min. The model $\widehat{AL}$ intensifications (many of which correspond to substorms) are more smoothed than the exponentially increasing [Weimer, 1991] and abruptly decreasing observed events; this shows that the model accounts mainly for the directly driven response. Details of substorm phases can only be reproduced by a nonlinear model which should include the second, unloading mode of magnetospheric response [Baker et al., 1990; Klimas et al., 1992].

Each one of the LRC parameters contains contributions from different parts of the magnetosphere. For instance, part of the effect of the inductance is due to the lag associated with the Alfven transit time from the dayside reconnection sites to the tail of the magnetosphere. An additional contribution to L comes from the change of flux across the tail [Klimas et al., 1992]. The "capacitive" property of the magnetosphere to store energy, is associated with the average kinetic energy of the plasma sheet particle distribution [McPherron, 1991]. The dissipative system loses its energy to several sinks, primarily the ionosphere and the current sheet, both of which are modeled by a constant resistor.

The RC and L/R time scales of the model are comparable to time scales of the impulse response function of AL with respect to B_{South}. The impulse response H relates most generally an input and a linear output

$$O(t) = \int_{-\infty}^{\infty} H(\tau) I(t-\tau) d\tau. \tag{2.3}$$

Linear prediction filter analysis solves for the response function H in the above equation when O(t) and I(t) are known. When O=AL and I=vB_{South} (v: the solar wind speed), the response H is a function of time which has a finite duration of about 2.5 h. This is a linear estimate of the "memory"

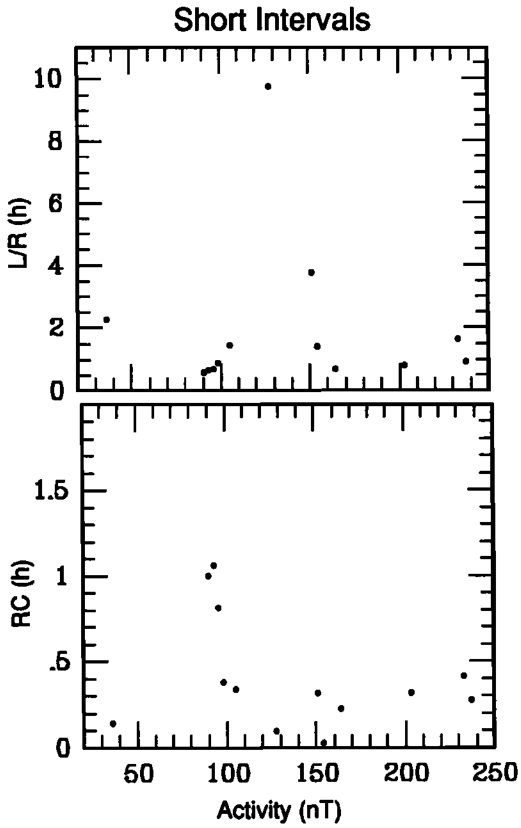

Figure 4 The L/R time scale corresponding to the overall length of the filter (upper panel) plotted againest the average AL activity. The RC time scale at which the response peaks (lower panel) drops for higher activities.

of the auroral geomagnetic response to recent inputs. The maxima of H are associated with the dominant contributions in the observed AL intensifications. The response has at least one maximum at 20 min after the input onset which is associated with the directly driven response; this time scale decreases for high activity intervals. There is often a second peak which can be as late as 1 h [Bargatze et al., 1985] and whose presence and location seem to vary systematically with the input activity. The second peak has been interpreted as representing the loading-unloading part of the response.

The LRC circuit has the single-peaked impulse response

$$H_{LRC}(t) \sim e^{-t/2RC} \sinh \left\{ \frac{t}{2RC} \left[1 - \frac{(2RC)^2}{LC} \right]^{1/2} \right\}. \quad (2.4)$$

The RC rise time shows a decrease with the AL time average (Fig. 4a). The peak of the response occurs at a time which is reached later for lower activity and earlier for higher activity; a similar decrease with activity is seen in the time of occurrence of the first peak in the linear prediction filter response [Bargatze et al., 1985]. The L/R decay time determines the length of the response and lies in the range of 1–2 h except for an apparent increase around 128 nT (Fig. 4b). However, subsequent studies (Vassiliadis, 1993; to appear) do not confirm this increase.

Although the circuit does not reproduce the high-frequency auroral magnetospheric variations, the correlation between prediction and observation implies that the system

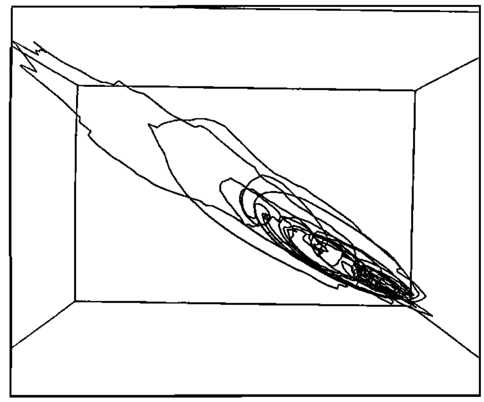

Figure 5 A three-dimensional trajectory whose components are a time series of the circuit's input B_{South} and two series of the model output (V_C and i_L). A very similar trajectory is obtained from one time series constructed from B_{South} and two constructed from the observed AL.

(2.1) captures a significant amount of the observed intensifications for longer time scales. For those intervals the coupling of the magnetosphere to the solar wind is modeled by a system with few degrees of freedom ("low dimensional"). Such a description is possible for a complex system when the system undergoes self-organization. Self-organization of complex systems persists under external driving and thus the auroral magnetospheric dynamics can maintain a state of self-organization while driven by the turbulent solar wind.

The empirical model variables (B_{South}, i_L, v_C) represent a reduced *state of the auroral geomagnetic system*. Additional variables are necessary for a higher-order model that includes unloading and a more complex input. The three variables can be visualized as coordinates in a state space (Fig. 5). Since i_L and v_C come from a low-pass filter (the LRC circuit) which does not resolve time scales shorter than RC = 0.5 h, the Southward component has also been smoothed by a running average of similar length. The structure in the 3–D state space is more clear and smooth than the state spaces constructed with "delay coordinates" [Vassiliadis et al., 1990; Roberts, 1991; Shan et al., 1991; Pavlos et al., 1992; Prichard and Price, 1992]. The structure is confined in a 2-D planar surface made up of loops which return close to the origin, the quiet geomagnetic conditions. A cut perpendicular to the plane of the structure (a "Poincare section") shows that the spatial distribution of the loops on the plane does not form a deterministic repeated pattern, but instead follows the intensifications of the random solar wind input. This phase space constructed from B_{South} and AL is very similar to the phase spaces composed either from the input or the output alone. The similarity stresses the dominance of the solar wind driving on the system dynamics so that the geomagnetic "output" is a function strongly dependent on the input.

Discussion

The relation of the solar wind flux input (represented by B_{South}) and the auroral magnetospheric activity (AL) was examined using a linear LRC circuit as a model for the response. The optimal LRC parameters for a given B_{South}-AL interval were determined by minimizing the squared difference between the observed activity and the model's output. This is similar to Weimer's [1991] study of substorm events and in contrast to previous studies of electrical circuit models [Rostoker and Bostroem, 1976; Liu et al., 1988; Rostoker and Pascal, 1990] which calculated the parameters from theoretical considerations of magnetospheric features. The model reproduces intervals of various levels of activity with relatively small variations in its parameters. The peak of the circuit's response function is seen to occur earlier as the average input level is increased, a decrease which was also observed in linear prediction filtering studies of the response [Bargatze et al., 1985]. A second time scale, related to the decay time of the filter response does not vary systematically, similarly to the length of the response function of prediction filters.

The correlation between observed and reproduced behavior shows that in many cases the dynamical behavior is consistent with a driven system of few effective degrees of freedom. Thus the LRC model provides an alternative approach to the effective degrees of freedom of the geomagnetic response by examining dynamical rather than statistical features of the time series data (such as the correlation dimension). It shows that a linear model can successfully reproduce the occurrence of active or quiet geomagnetic conditions in the auroral zone.

The model reproduces the driven response to AL while features such as the AL intensifications during expansion or the presence of unloading events without prior strong driving, must be described with nonlinear models [Klimas et al., 1992]. Also, by construction the model does not have the bimodal response of linear prediction filters. However, the equations (2.1) are the common basis of several nonlinear dynamical-system models of the response [Goertz et al., 1993; Klimas et al., 1992; Weimer, 1991] and as such they show the contribution of the driven linear component to the predictive ability of the models. Nonlinear driven models [e.g. Goertz et al., 1993] can reproduce geomagnetic fluctuations, but so far they have been tested with intervals of a dominant directly driven response. The future extension of the LRC circuit involves changing the resistance to a variable one, since the energy dissipation rate changes significantly and also because the R parameter has the highest amount of variation with the choice of interval. Nonlinear empirical models can serve as bases for theoretical models while they are of interest in forecasting since they are faster than large-scale simulations.

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